College of Engineering and Management, Kolaghat

Signals and Systems

Lec-4 &5

Prof. Anup Gorai

Classification of
Continuous Time- Signals
and Discrete Time Signals:

1. Energy Signals and Power Signals

2. Periodic and aperiodic (non-periodic) Signals

3. Even and odd (or Symmetric and Anti-symmetric) Signals

4. Causal and Non-causal Signals

Energy Signals and Power Signals

➢ For continuous-time signal:

- Let consider an arbitrary continuous-time signal , x(t).

- The normalized energy content E of x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

- The normalized average power P of x(t) is defined as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

➢ For discrete-time signal:

- Let consider a discrete-time signal x[n]
- The Energy of the discrete –time signal is E defined as

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- The normalized average power P of x[n] is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Energy Signals:

❑ x(t) (or x[n]) is said to be an energy signal (or sequence) if and only if

i.) $0 < E < \infty$ (Finite), and

ii.) $P = 0$.

Power Signals:

❑ x(t) (or x[n]) is said to be a power signal (or sequence) if and only if

i.) $0 < P < \infty$ (Finite), and

ii.) $E = \infty$.

If above conditions are not satisfied, then signal is neither energy signal nor power signal.

Problems: Find whether the following signals are Power, Energy or neither energy nor power signals.

(a) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

(b) $x(t) = tu(t)$

(c) $x(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}$

(d) u(n+2)-u(n-2)

(a) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

The energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{3}\right)^n\right]^2 \qquad \left[\begin{aligned} u(n) &= 1 \text{ for } n \geq 0 \\ &= 0 \text{ for } n < 0 \end{aligned}\right]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{1}{1-\frac{1}{9}} \qquad \left[1 + a + a^2 + \cdots = \frac{1}{1-a}\right]$$

$$= \frac{8}{9}$$

The Power $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$

$= \lim_{N \to \infty} \frac{1}{2N+1} \cdot \left[\frac{1 - \left(\frac{1}{9}\right)^{N+1}}{1 - \frac{1}{9}} \right]$

$= 0$

So , Given signal is an Energy Signal.

Formula:

$$\left[1 + a + a^2 + \cdots + a^n = \frac{1 - a^{n+1}}{1 - a} \right]$$

if $a < 1$

$$\left|e^{j(\omega n + \theta)}\right| = 1$$

$$\left|e^{j\omega n}\right| = 1$$

(b)

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T/2} t^2 dt = \lim_{T\to\infty} \frac{(T/2)^3}{3} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} t^2 dt = \lim_{T\to\infty} \frac{1}{T} \frac{(T/2)^3}{3} = \infty$$

Thus, signal x(t) is neither an energy signal nor a power signal.

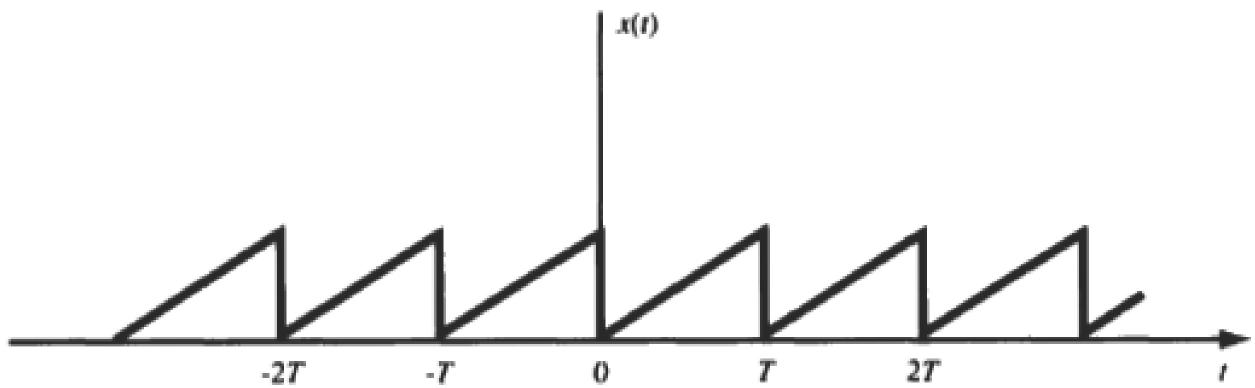
Periodic and aperiodic (non-periodic) Signals

- <u>For continuous-time signal:</u>

A continuous-time signal x (t) is said to be periodic with **period T** if there is a positive nonzero value of T for which

$$x(t + T) = x(t) \quad \text{for } \textit{all } t$$

<u>Example:</u>



Above signal follows the $x(t + mT) = x(t)$ for all t and any integer m.

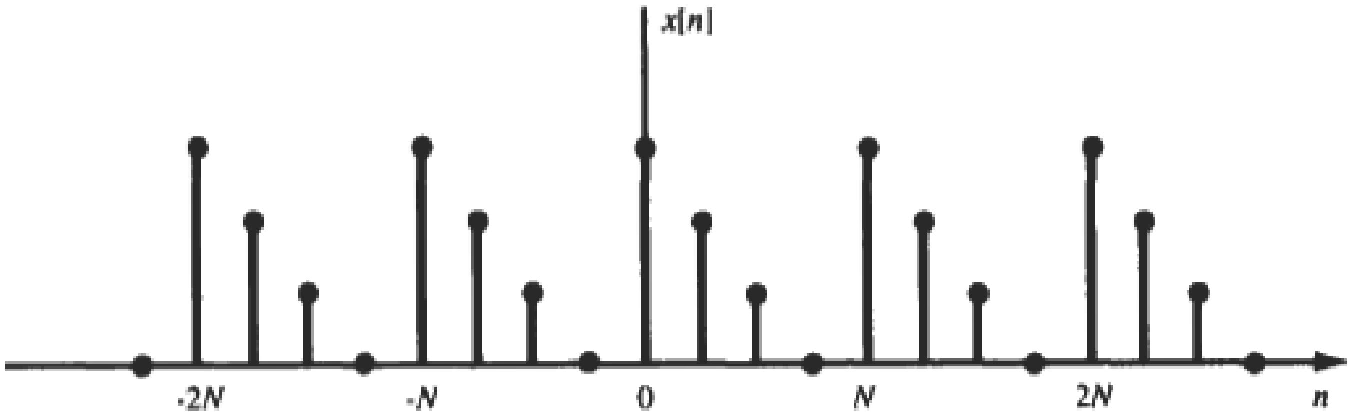
<u>Fundamental period (T_0):</u> fundamental period T_0, of x (t) is the smallest positive value of T.

➢For discrete-time signal:

Periodic discrete-time signals are defined analogously.
A sequence (discrete-time signal) $x[n]$ is periodic with period N if there is a positive integer N for which

$$x[n + N] = x[n] \text{ for all } n$$

Example:



Above signal follows that $x[n + mN] = x[n]$ for all n and any integer m.

Here N_0 is the fundamental period of the discrete time signal.

##Any signal or sequence which is not periodic is called a nonperiodic (or aperiodic) sequence.

Problems:

Let consider a signal $x(n)=A\,\mathrm{Sin}(\omega_0 n+\theta)$.
Signal is periodic if satisfy the following condition $x(n+N)=x(n)$.

So, $x(n+N)= A\,\mathrm{Sin}(\omega_0 (n+N)+\theta) = A\,\mathrm{Sin}(\omega_0 n+ \omega_0 N+\theta)$

Now, $\omega_0 N=2\pi m$, where m is the positive integer.
Or, **$N=2\pi(m/ \omega_0)$**

Problem 1:

(1) $x(n) = e^{j6\pi n}$

$\omega_0 = 6\pi$

The fundamental frequency is multiple of π. ~~That~~ Therefore, the signal is periodic.

$N = 2\pi\left(\frac{m}{\omega_0}\right)$

$= 2\pi\cdot\left(\frac{m}{6\pi}\right)$

$= \frac{m}{3}$

$= 1$ | $m = 3$

So, fundamental period $= 1$

② $x(n) = e^{j\frac{3}{5}\left(n+\frac{1}{2}\right)}$,

So, $\omega_0 = \frac{3}{5}$, which is not a multiple of π, so Signal is aperiodic.

③ $x(n) = \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{3\pi}{4}\right)n$

$\Rightarrow \omega_1 = \frac{\pi}{3}$ and $\omega_2 = \frac{3}{4}\pi$

both the fundamental frequencies are multiple of π, so signals are periodic.

Fundamental period of $\cos\left(\frac{\pi}{3}n\right)$,

$$\boxed{N_1} = 2\pi\left(\frac{m}{\omega_1}\right) = 2\pi\left(\frac{m}{\pi/3}\right)$$

$$= 6m \quad \text{if } m=1$$

$$= 6$$

$$N_2 = 2\pi\left(\frac{m}{\frac{3\pi}{4}}\right) = \frac{8m}{3} = 8 \quad \text{if } m=3$$

L.C.M

$$2\,\underline{|\,6,\ 8}$$
$$\quad 3,\ 4$$

$$\boxed{N = 24}$$

Even and odd (or Symmetric and Anti-symmetric) Signals

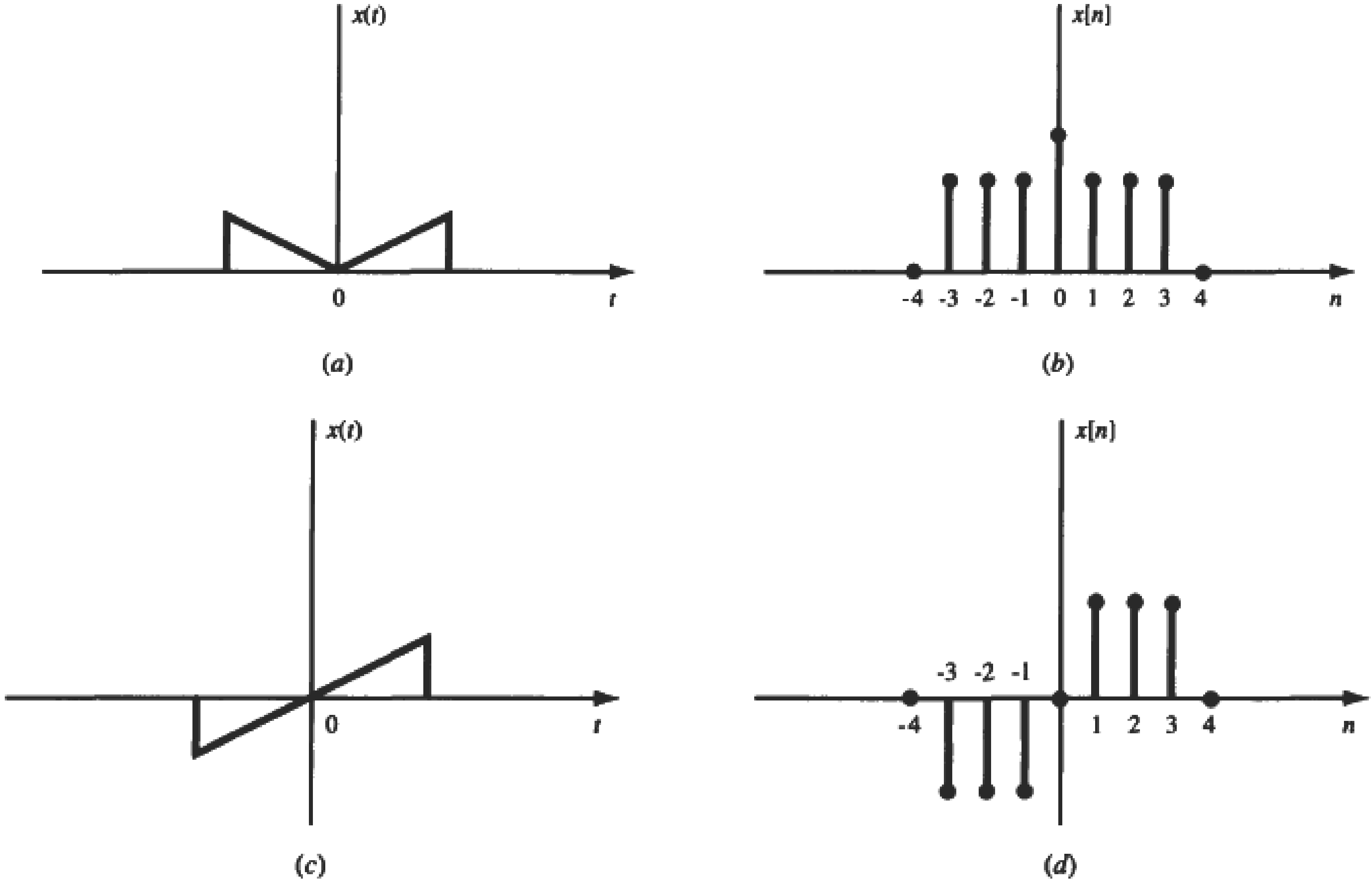
- Signal x (t) or x[n] is referred to as an even or symmetric signal if

$x(-t) = x(t)$ (continuous time-signal)
$x[-n] = x[n]$ (Discrete time signal)

- A signal x (t) or x[n] is referred to as an odd or anti-symmetric signal if

$x(-t) = -x(t)$ (continuous time-signal)
$x[-n] = -x[n]$ (Discrete time signal)

Examples:



Figs.: Examples of even signals **(a and *b*)** *and odd signals* ***(c and d).***

- Any signal $x(t)$ or $x[n]$ can be expressed as a sum of two signals, one of which is even and one of which is odd.

$x(t)= x_e(t)+x_o(t)$ .... (1)

and $x(-t)= x_e(-t)+x_o(-t) = x_e(t)-x_o(t)$ ....(2)

Now adding eqs. (1)and (2)

$2x_e(t)= x(t)+x(-t)$

Or $x_e(t)= (1/2)\{x(t)+x(-t)\}$ ....(3)

And $x_o(t)= (1/2)\{x(t)-x(-t)\}$ ....(4)

- Similarly for discrete time signals,

$x_e[n] = (1/2)\{x[n]+x[-n]\}$ ....(5)

And $x_o[n] = (1/2)\{x[n]-x[-n]\}$ ....(6)

##Note that the product of two even signals or of two odd signals is an even signal and that the product of an even signal and an odd signal is an odd signal.

Causal and Non-causal Signals

The signal x[n] (or x(t)) is said to be **causal** if its value is zero for n< 0 (or t<0),
otherwise the signal is **non-causal**.

~~Exples~~ Examples →

Causal

$x_1(n) = a^n u(n)$

$x_2(n) = \{1, 2, 0, 5, 2, 4\}$

$x_3(n) = \sum_{n=0}^{\infty} |e^{an}|$

Non-causal

$x_1(n) = a^n u(-n+1)$

$x_2(n) = \{1, 2, 0, 1, -1\}$

CLASSIFICATION OF SYSTEMS

1. **Continuous -time Systems:**

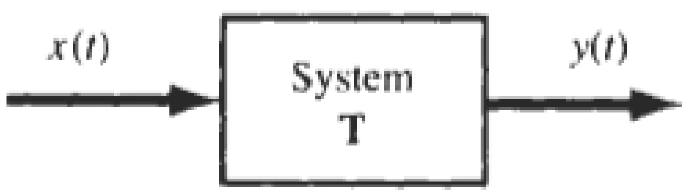
If the input x(t) and output y(t) signals are continuous-time signals, then the system is called a continuous-time system**.** A continuous-time system is shown in fig. (a).

$y(t)=T[x(t)]$

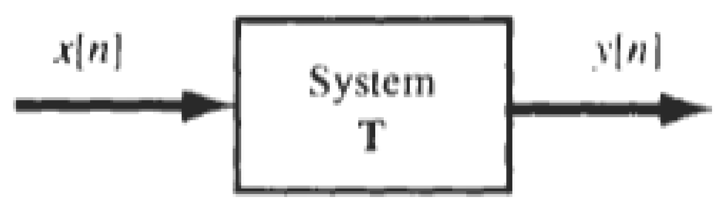
2.Discrete-Time Systems:

If the input (x[n]) and output (y[n]) signals are discrete-time signals or sequences, then the system is called a discrete-time system. Fig. (b) shows a discrete-time system.

$y[n]=T[x[n]]$



(a)



(b)

❑Classification of the continuous-time systems and discrete-time systems:

1. Static and Dynamic System
2. Causal and Non-Causal Systems
3. Linear and Non-linear systems
4. Time-variant and Time-invariant Systems
5. FIR and IIR systems
6. Stable and unstable Systems

1. Static and Dynamic system

A system is said to be **static or memoryless** if the output at any time depends on only the input at that same time. Otherwise, the system is said to **have memory or Dynamic system**.

In discrete time systems , system is said to be static if the output at any instant (n) depends on the input samples at the same time, but do not on past or future samples of the input.

Examples:

Continuous –time systems:

Static: y(t)= R x(t)

Dynamic: $$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau$$

Discrete – time systems:

Static:
$y(n)=ax(n)$
$y(n)= ax^2(n)$

Dynamic:
$y(n)= x(n-1) + x(n-2)$
$y(n)=x(n+1)+x(n+2)$

$y(n)=ax(n)$ if $n=0$ then $y(0)=ax(0)$ or if $n=1$, then $y(1)=ax(1)$ or if $n=-1$, then $y(-1)=ax(-1)$

$y(n)= ax^2(n)$ if $n=0$ then $y(0)= ax^2(0)$ or if $n=-1$ then $y(-1)= ax^2(-1)$ or if $n=1$ then $y(1)= ax^2(1)$

$y(n)= x(n-1) + x(n-2)$ if $n=0$ then $y(0)= x(-1) + x(-2)$ or if $n=1$ then $y(n)= x(0) + x(1)$

$y(n)=x(n+1)+x(n+2)$ if $n=0$ then $y(0)=x(1)+x(2)$ or if $n=1$ then $y(1)=x(2)+x(3)$

2. Causal and Non-causal Systems

A system is called **causal** *if its output y (t) at an arbitrary time $t = t_0$ depends on only* the input *x (t) for t ≤0.*

That is, the output of a causal system at the present time depends on only the present and/or past values of the input, not on its future values.

A system is called **non-causal** *if it is not causal.*

Examples:

Causal

$$y(t)= x(t) + x(t-1)$$

$$y(n)=x(n-1)+ x^2(n)$$

Non-causal

$$y[n]=x[n+1]$$

$$y[n]=x[n+1]+x[n]$$

$y(t)= x(t) + x(t-1)$

If $t=0$ then

$y(0)=x(0)+x(-1)$

if $t=1$ then

$y(1)=x(1)+x(0)$

$y[n]=x[n+1]+x[n]$

If $n=0$ then

$y[0]=x[1]+x[0]$

If $n=1$ then

$y[1]=x[2]+x[1]$

3. Linear and Nonlinear Systems

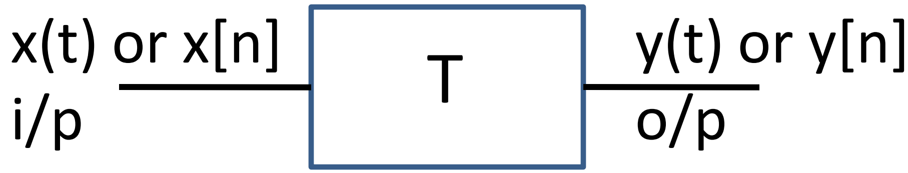


Fig.1: System

$$y(t)=T[x(t)] \quad \text{Or} \quad y[n]=T[x[n]] \qquad \text{.....(1)}$$

In fig.1, if x(t) or x[n]= x1 then corresponding output will be y(t) or y[n]=y1 and if x(t) or x[n]= x2 then corresponding output will be y(t) or y[n]=y2.

If the operator T in Eq. (1) satisfies the following two conditions, then T is called a linear operator and the system represented by a linear operator T is called a linear system:

Condition 1: **Additivity**

Given that T[x1]= y1 and T[x2] = y2, then
T[x1+x2] =y1+y2 .... (2)

for any signals x1 and x2.

Condition 2: **Homogeneity (or Scaling)**

If we multiply any scalar (a) with the input signal (x) , then
T[ax]=ay. ....(3)

Any system that does not satisfy Eq. (2) and/or Eq. (3) is classified as a nonlinear system.

Equations (2) and (3) can be combined into a single condition as $T[a_1x1+a_2x2] =a_1y1+a_2y2$ ....(4)
where a_1 and a_2 are arbitrary scalars.

Above equation is also known as the superposition property.

For Linear continuous-time system:

$$T[a_1x1(t)+a_2x2(t)] = a_1y1(t)+a_2y2(t) = a_1T[x1(t)]+a_2T[x2(t)] \quad \text{.................} \; (5)$$

For Linear discrete-time system:

$$T[a_1x1[n]+a_2x2[n]] = a_1y1[n]+a_2y2[n] = a_1T[x1[n]]+a_2T[x2[n]] \quad \text{.................} \; (6)$$

Examples:

So, system is Nonlinear system.

① $y(n) = x^2(n)$

outputs due to signals $x_1(n)$ and $x_2(n)$

$\therefore y_1(n) = T[x_1(n)] = x_1^2(n)$

$y_2(n) = T[x_2(n)] = x_2^2(n)$

Weighted Sum of outputs

$a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 x_1^2(n) + a_2 x_2^2(n)$

output due to weighted sum of inputs **

weighted sum of the input signals

$= a_1 x_1(n) + a_2 x_2(n)$

$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)] = [a_1 x_1(n) + a_2 x_2(n)]^2$

(2) $y(n) = n x(n)$

→ outputs due to signals $x_1(n)$ and $x_2(n)$

$$\therefore y_1(n) = T[x_1(n)] = n x_1(n)$$

$$y_2(n) = T[x_2(n)] = n x_2(n)$$

→ weighted sum of outputs $y_3(n) =$

$$a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$= a_1 n x_1(n) + a_2 n x_2(n)$$

→ weighted sum of the input signals

$$= a_1 x_1(n) + a_2 x_2(n)$$

→ output due to the weighted sum of inputs

$$y_4(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$

$$= n a_1 x_1(n) + n a_2 x_2(n)$$

$$\therefore \boxed{y_3(n) = y_4(n)}$$

So system is Linear

4. Time variant and Time-Invariant Systems

A system is said to be time-invariant, if characteristics of the system do not change with time.

To test if any given system is time-invariant, first apply an arbitrary sequence $x(n)$ and find $y(n)$.

Now delay the input sequence by k samples and find output sequence, and denote it as $y(n,k) = T[x(n-k)]$

Next Delay the output sequence by k samples, denote it as $y(n-k)$.

If $y(n,k) = y(n-k)$, system is time invariant.

Examples:

$y(n)=x(n)+x(n-1)$

- Given $y(n)=T[x(n)]=x(n)+x(n-1)$

Step 1:

Delaying the input of the given system by k samples

$y(n,k)=T[x(n-k)]=x(n-k)+x(n-k-1)$ ....(1)

Step 2:

Delaying the output of the given system by k samples

So, $y(n-k)=x(n-k)+x(n-k-1)$ ....(2)

From the eqs. (1) and (2)

$y(n,k)=y(n-k)$

So, system is time-invariant.

Assignment:

1. Show that

(a) $x[n] * \delta[n] = x[n]$

(b) $x[n] * \delta[n - n_0] = x[n - n_0]$

2. Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

(a) $x(t) = \cos\left(t + \frac{\pi}{4}\right)$

(b) $x(t) = \sin \frac{2\pi}{3} t$

(c) $x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t$

(d) $x(t) = \cos t + \sin \sqrt{2}\, t$

(e) $x(t) = \sin^2 t$

(f) $x(t) = e^{j[(\pi/2)t - 1]}$

(g) $x[n] = e^{j(\pi/4)n}$

(h) $x[n] = \cos \frac{1}{4} n$

(i) $x[n] = \cos \frac{\pi}{3} n + \sin \frac{\pi}{4} n$

(j) $x[n] = \cos^2 \frac{\pi}{8} n$

3. Determine whether the following signals are energy signals, power signals, or neither.

(a) $x(t) = e^{-at}u(t), \quad a > 0$

(b) $x(t) = A\cos(\omega_0 t + \theta)$

(c) $x(t) = tu(t)$

(d) $x[n] = (-0.5)^n u[n]$

(e) $x[n] = u[n]$

(f) $x[n] = 2e^{j3n}$